Chapter 3. The quantum and electromagnetic forms of electron theory

1.0. Introduction

In the chapter 2 we have shown that the Dirac electron equation is the equation of EM wave, moving along a ring trajectory. Thus, the difference between two forms – quantum and electromagnetic - consists only in the mathematical form of record: the complex form of the EM equations corresponds to the operationally-matrix form of the quantum equations.

The Dirac electron theory has a lot of particularities. In the modern interpretation these particularities are considered as mathematical features that do not have a physical meaning. The electromagnetic form of part of them we have considered in the chapter 2. On the basis of the chapter 2 we will show also that all other mathematical particularities of the Dirac electron theory have the known electrodynamics sense.

2.0. Electrodynamics meaning of the forms of the Dirac equations

2.2. The quantum Dirac equation forms with mass

There are two bispinor Dirac equations (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955; Fermi, 1960) (the description of the equation characteristics and parameters see in the chapter 2):

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\, mc^2\right]\psi = 0\,, \tag{1.1}$$

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) - \hat{\beta}\, mc^2\right]\psi = 0\,, \tag{1.2}$$

which correspond to two signs of the relativistic expression of the electron energy:

$$\varepsilon = \pm\sqrt{c^2\vec{p}^{\,2} + m^2c^4}\,, \tag{1.3}$$

but for each sign of the expretion (1.3) there are two Hermitian-conjugate Dirac equations. Thus there are two Hermitian-conjugate equations, corresponding to the minus sign of the expression (1.3):

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\, mc^2\right]\psi = 0\,, \tag{1.4'}$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\, mc^2\right] = 0\,, \tag{1.4''}$$

and two equations that correspond to plus signs of (1.3):

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) - \hat{\beta}\ mc^2\right]\psi = 0\,, \qquad (1.5')$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) - \hat{\beta}\ mc^2\right] = 0\,, \qquad (1.5'')$$

We will use further the wave function in the matrix form of the plane EM wave, moving as in the chapter 2 along y - axis:

$$\psi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix},\ \psi^+ = \begin{pmatrix} E_x & E_z & -iH_x & -iH_z \end{pmatrix}, \qquad (1.6)$$

which with the following choice of the Dirac matrices

$$\hat{\alpha}_0 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & \hat{\sigma}_0 \end{pmatrix},\ \hat{\vec{\alpha}} = \begin{pmatrix} 0 & \hat{\vec{\sigma}} \\ \hat{\vec{\sigma}} & 0 \end{pmatrix},\ \hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix}, \qquad (1.7)$$

where $\hat{\vec{\sigma}}$ are Pauli spin matrices, give the correct electrodynamics expressions.

2.2. The EM Dirac equation forms

Let us consider first two Hermitian-conjugate equations, corresponding to the minus sign of the expression (1.3). Using (1.6), from (1.4') and (1.4'') we obtain:

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -\vec{j}^e_x \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = \vec{j}^m_z \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = -\vec{j}^e_z \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = \vec{j}^m_x \end{cases},\ (1.7'),\ \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = \vec{j}^e_x \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = -\vec{j}^m_z \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = \vec{j}^e_z \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = -\vec{j}^m_x \end{cases},\ (1.7'')$$

where

$$\vec{j}^e = i\frac{\omega_e}{4\pi}\vec{E} = i\frac{1}{4\pi}\frac{c}{r_e}\vec{E}\,, \qquad (1.8')$$

$$\vec{j}^m = i\frac{\omega_e}{4\pi}\vec{H} = i\frac{1}{4\pi}\frac{c}{r_e}\vec{H}\,, \qquad (1.8'')$$

are the complex currents, in which $\omega_e = \dfrac{2mc^2}{\hbar}$, and $r_e = \dfrac{\hbar}{2mc}$. Thus, the equations (1.4') and (1.4'') are Maxwell equations with complex currents. As we see, *the Hermitian-conjugate equations* (1.7) *and* (1.8*) differ by the current directions*.

Let us consider now the equations that correspond to plus signs of (1.3). The electromagnetic form of the equation (1.5') is:

$$\begin{cases} \dfrac{1}{c}\dfrac{\partial E_x}{\partial t} + \dfrac{\partial H_z}{\partial y} = -\vec{j}_x^{\,e} \\ \dfrac{1}{c}\dfrac{\partial H_z}{\partial t} + \dfrac{\partial E_x}{\partial y} = \vec{j}_z^{\,m} \\ \dfrac{1}{c}\dfrac{\partial E_z}{\partial t} - \dfrac{\partial H_x}{\partial y} = -\vec{j}_z^{\,e} \\ \dfrac{1}{c}\dfrac{\partial H_x}{\partial t} - \dfrac{\partial E_z}{\partial y} = \vec{j}_x^{\,m} \end{cases}, \qquad (1.9)$$

Obviously, the electromagnetic form of the equation (1.5'') will have the opposite signs of the currents comparatively to (1.9).

Comparing (1.9) and (1.7) we can see that the *equation* (1.9) *can be considered as the Maxwell equation of the retarded wave*. If we don't want to use the retarded wave, we can transform the wave function of the retarded wave to the form:

$$\psi_{ret} = \begin{pmatrix} E_x \\ -E_z \\ iH_x \\ -iH_z \end{pmatrix}, \qquad (1.10)$$

Then, contrary to the system (1.9) we get the system (1.8). The transformation of the function ψ_{ret} to the function ψ_{adv} is called the charge conjugation operation.

Note that the electron and positron wave functions can be considered as the retarded and advanced waves. So the above result links also with the theory of advanced waves of Wheeler and Feynman (Wheeler and Feynman, 1945; Wheeler, 1957). (See also Dirac's work on time-symmetric classical electrodynamics (Dirac, 1938), and about this theme - Konopinski's book (Konopinski, 1980).

3.0. Electrodynamics meaning of the bispinor forms

It is known that there are 16 Dirac matrices of 4x4 dimensions. We use the set of matrices which used Dirac himself and we will name it α -set (1.4).

It can be shown that the tensor dimension of bilinear form follows from its nonlinear electrodynamics forms. Enumerate corresponding Dirac's matrices (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955):

1) $\hat{\alpha}_4 \equiv \hat{\beta}$, (3.1')

2) $\hat{\alpha}_\mu = \left\{\hat{\alpha}_0, \hat{\vec{\alpha}}\right\} \equiv \left\{\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4\right\}$, (3.1'')

3) $\hat{\alpha}_5 = \hat{\alpha}_1 \cdot \hat{\alpha}_2 \cdot \hat{\alpha}_3 \cdot \hat{\alpha}_4$, (3.1''')

4) $\hat{\alpha}_\mu^A = \hat{\alpha}_5 \cdot \hat{\alpha}_\mu$, (3.1'''')

5) $$\hat{\alpha}_{\mu\nu} = -\hat{\alpha}_{\nu\mu} = \begin{cases} i\hat{\alpha}_\nu \hat{\beta}\ \hat{\alpha}_\mu, & \mu \neq \nu \\ 0, & \mu = \nu \end{cases}, \quad (3.2)$$

where 1) scalar, 2) 4-vector, 3) pseudoscalar, 4) 4-pseudovector, 5) antisymmetrical tensor of second rank are.

Let's calculate electrodynamics values corresponding to these matrices: $O = \psi^+ \hat{\alpha}\ \psi$, where ψ is given by (1.6):

1) $\psi^+ \hat{\alpha}_4\ \psi = \left(E_x^2 + E_z^2\right) - \left(H_x^2 + H_z^2\right) = \vec{E}^2 - \vec{H}^2 = 8\pi\ I_1$, where I_1 is the *first scalar* (invariant) of Maxwell theory, i.e. the Lagrangian of electromagnetic field in vacuum;

2) $\psi^+ \hat{\alpha}_o \psi = \vec{E}^2 + \vec{H}^2 = 8\pi\ U$, where U is the *energy density* of electromagnetic field;

$\psi^+ \hat{\alpha}_y \psi = -\dfrac{8\pi}{c} \vec{S}_{Py} = -8\pi\ c\vec{g}_y$, where $\vec{g}_y$ is the *momentum density* of the electromagnetic wave field moved along the *Y*-axis. As it is known, the value $\left\{\dfrac{1}{c}U, \vec{g}\right\}$ is 4-vector of energy-momentum.

3) $\psi^+ \hat{\alpha}_5\ \psi = 2\ \left(E_x H_x + E_z H_z\right) = 2\ \left(\vec{E} \cdot \vec{H}\right)$, which is the *pseudoscalar* of electromagnetic field, and $\left(\vec{E} \cdot \vec{H}\right)^2 = I_2$ is the second scalar (invariant) of electromagnetic field theory.

4) $\psi^+ \hat{\alpha}_5 \hat{\alpha}_0 \psi = 2\left(E_x H_x + E_z H_z\right) = 2\left(\vec{E} \cdot \vec{H}\right)$

$$\psi^+\hat{\alpha}_5\hat{\alpha}_1\psi = -2i\left(E_x E_z - H_x H_z\right),$$
$$\psi^+\hat{\alpha}_5\hat{\alpha}_2\psi = 0,$$
$$\psi^+\hat{\alpha}_5\hat{\alpha}_3\psi = -i(E_x{}^2 - E_z{}^2 - H_x{}^2 + H_z{}^2).$$

As we will show in the chapter 5, the 4-pseudovector is connected with spirality of particles.

5) Tensor $\psi^+\hat{\alpha}_{\mu\nu}\psi$ we can write in compact form:

$$\left(\alpha_{\mu\nu}\right)=$$
$$\begin{pmatrix} 0 & E_x{}^2 - E_z{}^2 + H_x{}^2 - H_z{}^2 & 0 & -2\left(E_x H_z + E_z H_x\right) \\ -\left(E_x{}^2 - E_z{}^2 - H_x{}^2 + H_z{}^2\right) & 0 & 2\left(E_x E_z - H_x H_z\right) & 0 \\ 0 & -2\left(E_x E_z - H_x H_z\right) & 0 & -2\left(E_x H_x - E_z H_z\right) \\ 2\left(E_x H_z + E_z H_x\right) & 0 & 2\left(E_x H_x - E_z H_z\right) & 0 \end{pmatrix}$$

As we will show below this thensor defines the Lorentz force.

4.0. About statistical interpretation of the wave function

As it is known, from the Dirac equation the probability continuity equation can be obtained (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955; Fermi, 1960):

$$\frac{\partial\ P_{pr}\left(\vec{r},t\right)}{\partial\ t} + div\ \vec{S}_{pr}\left(\vec{r},t\right) = 0, \tag{4.1}$$

Here $P_{pr}\left(\vec{r},t\right) = \psi^+\hat{\alpha}_0\ \psi$ is the probability density, and $\vec{S}_{pr}\left(\vec{r},t\right) = -c\,\psi^+\hat{\vec{\alpha}}\ \psi$ is the probability flux density. Using the above results we can obtain: $P_{pr}\left(\vec{r},t\right) = 8\pi\ U$ and $\vec{S}_{pr} = c^2\vec{g} = 8\pi\ \vec{S}$. Then the electromagnetic form of the equation (3.15) is:

$$\frac{\partial\ U}{\partial\ t} + div\ \vec{S} = 0\,, \tag{4.2}$$

which is the form of *energy-momentum conservation law* of the EM field.

5.0. The electrodynamics meaning of the matrices choice

According to Fermi (Fermi, 1960) "it can prove that all the physical consequences of Dirac's equation do not depend on the special choice of Dirac's matrices… In particular it is possible to interchange the roles of the four matrices by unitary transformation. So, their differences are only apparent".

The matrix sequence $\left(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3\right)$ agrees with the electromagnetic wave, which has $-y$-direction. A question arises: how to describe the waves, which have

x and z - directions? Introducing the axes' indexes, which indicate the electromagnetic wave direction, we can write three groups of the matrices, each of which corresponds to one and only one wave direction:

$$(\hat{\alpha}_{1x}, \hat{\alpha}_{2y}, \alpha_{3z}),\ (\hat{\alpha}_{2x}, \hat{\alpha}_{3y}, \hat{\alpha}_{1z},),\ (\hat{\alpha}_{2z}, \hat{\alpha}_{1y}, \hat{\alpha}_{3x}).$$

Let us choose now the wave function forms, which give the correct Maxwell equations for the x and z - directions. Taking into account (1.6) as the initial form of the $-y$ - direction, from it, by means of the indexes' transposition around the circle (see. Fig. 1), we will get other forms.

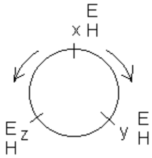

Fig. 1

Since in this case the Poynting vector has the minus sign, we can suppose that the transposition must be counterclockwise. Let us examine this supposition, checking the Poynting vector values:

The sets $(\hat{\alpha}_{1x}, \hat{\alpha}_{2y}, \alpha_{3z})$, $(\hat{\alpha}_{2x}, \hat{\alpha}_{3y}, \hat{\alpha}_{1z},)$, $(\hat{\alpha}_{2z}, \hat{\alpha}_{1y}, \hat{\alpha}_{3x})$ correspond to the wave functions

$$\psi(y) = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix},\ \psi(x) = \begin{pmatrix} E_z \\ E_y \\ iH_z \\ iH_y \end{pmatrix},\ \psi(z) = \begin{pmatrix} E_y \\ E_x \\ iH_y \\ iH_x \end{pmatrix}$$

and to non-zero Poynting vectors

$$\psi^{+}\hat{\alpha}_{2_y}\psi = -2\left[\vec{E} \times \vec{H}\right]_y,$$

$$\psi^{+}\hat{\alpha}_{2x}\psi = -2\left[\vec{E} \times \vec{H}\right]_x, \psi^{+}\hat{\alpha}_{2z}\psi = -2\left[\vec{E} \times \vec{H}\right]_z$$ respectively.

As we see, we took the correct result. We can suppose now that by the clockwise indexes' transposition of the wave functions will describe the electromagnetic waves, which move in a positive direction along the co-ordinate axes. Let us prove this:

The sets $(\hat{\alpha}_{1x}, \hat{\alpha}_{2y}, \alpha_{3z})$, $(\hat{\alpha}_{2x}, \hat{\alpha}_{3y}, \hat{\alpha}_{1z})$, $(\hat{\alpha}_{2z}, \hat{\alpha}_{1y}, \hat{\alpha}_{3x})$ correspond to the wave functions

$$\psi(y)=\begin{pmatrix}E_z\\E_x\\iH_z\\iH_x\end{pmatrix},\ \psi(x)=\begin{pmatrix}E_y\\E_z\\iH_y\\iH_z\end{pmatrix},\ \psi(z)=\begin{pmatrix}E_x\\E_y\\iH_x\\iH_y\end{pmatrix}$$

and to non-zero Poynting vectors

$$\psi^+\hat{\alpha}_{2_y}\psi=2\left[\vec{E}\times\vec{H}\right]_y,\ \psi^+\hat{\alpha}_{2x}\psi=2\left[\vec{E}\times\vec{H}\right]_x,\ \psi^+\hat{\alpha}_{2z}\psi=2\left[\vec{E}\times\vec{H}\right]_z$$

respectively. As we see, once again we get the correct results.

Now we will prove that the above choice of the matrices and wave functions gives the correct electromagnetic equation forms. Using for example equation (1.5') and transposing the indexes clockwise we obtain for the positive direction of the electromagnetic wave the following results for x, y, z -directions correspondingly:

$$\begin{cases}\frac{1}{c}\frac{\partial E_y}{\partial t}+\frac{\partial H_z}{\partial x}=-j_y^e\\ \frac{1}{c}\frac{\partial H_z}{\partial t}+\frac{\partial E_y}{\partial x}=j_z^m\\ \frac{1}{c}\frac{\partial E_z}{\partial t}-\frac{\partial H_y}{\partial x}=-j_z^e\\ \frac{1}{c}\frac{\partial H_y}{\partial t}-\frac{\partial E_z}{\partial x}=j_y^m\end{cases},\ \begin{cases}\frac{1}{c}\frac{\partial E_z}{\partial t}+\frac{\partial H_x}{\partial x}=-j_z^e\\ \frac{1}{c}\frac{\partial H_x}{\partial t}+\frac{\partial E_z}{\partial x}=j_x^m\\ \frac{1}{c}\frac{\partial E_x}{\partial t}-\frac{\partial H_z}{\partial x}=-j_x^e\\ \frac{1}{c}\frac{\partial H_z}{\partial t}-\frac{\partial E_x}{\partial x}=j_z^m\end{cases},\ \begin{cases}\frac{1}{c}\frac{\partial E_x}{\partial t}+\frac{\partial H_y}{\partial x}=-j_x^e\\ \frac{1}{c}\frac{\partial H_y}{\partial t}+\frac{\partial E_x}{\partial x}=j_y^m\\ \frac{1}{c}\frac{\partial E_y}{\partial t}-\frac{\partial H_x}{\partial x}=-j_y^e\\ \frac{1}{c}\frac{\partial H_x}{\partial t}-\frac{\partial E_y}{\partial x}=j_x^m\end{cases},\quad (5.1)$$

As we can see, we have obtained three equation groups, each of which contains four equations, *as is necessary for the description of all electromagnetic wave directions*. In the same way for all other forms of the Dirac equation analogue results can be obtained.

Obviously, it is also possible via canonical transformations to choose the Dirac matrices in such a way that the electromagnetic wave will have any direction. Let us show it.

5.1. The EM meaning of canonical transformations of Dirac's matrices and bispinors

The choice (1.7) of the matrices is not unique (Akhiezer and Berestetskii, 1965; Schiff, 1955; Fock, 1932). As it is known, there is a free transformation of a kind: $\alpha = S\ a' S^+$, where S is a unitary matrix, called the canonical transformation operator and also the wave functions ψ' transformation $\psi = S\ \psi'$, which does not change the results of the theory.

If we choose matrices α' as:

$$\hat{\vec{\alpha}}_1 = \begin{pmatrix} \hat{\sigma}_x & 0 \\ 0 & \hat{\sigma}_x \end{pmatrix}, \hat{\vec{\alpha}}_1 = \begin{pmatrix} \hat{\sigma}_y & 0 \\ 0 & -\hat{\sigma}_y \end{pmatrix}, \hat{\vec{\alpha}}_3 = \begin{pmatrix} \hat{\sigma}_z & 0 \\ 0 & \hat{\sigma}_z \end{pmatrix}, \hat{\vec{\alpha}}_4 = \begin{pmatrix} 0 & -i\hat{\sigma}_y \\ i\hat{\sigma}_y & 0 \end{pmatrix}, \qquad (5.2)$$

then the functions ψ will be connected to functions ψ' according to the relationships:

$$\psi_1 = \frac{\psi'_1 - \psi'_4}{\sqrt{2}}, \quad \psi_2 = \frac{\psi'_2 + \psi'_3}{\sqrt{2}}, \quad \psi_3 = \frac{\psi'_1 + \psi'_4}{\sqrt{2}}, \quad \psi_4 = \frac{\psi'_2 - \psi'_3}{\sqrt{2}}, \quad (5.3)$$

The unitary matrix S, which corresponds to this transformation, is equal to:

$$S = \frac{1}{\sqrt{2}} \begin{Bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{Bmatrix}, \qquad (5.4)$$

It is not difficult to check that by means of this transformation we will also receive the equations of the Maxwell theory. Actually, using (1.6) and (5.3) it is easy to receive:

$$\frac{\psi'_1 - \psi'_4}{\sqrt{2}} = E_x, \frac{\psi'_2 + \psi'_3}{\sqrt{2}} = E_z, \frac{\psi'_1 + \psi'_4}{\sqrt{2}} = iH_x, \frac{\psi'_2 - \psi'_3}{\sqrt{2}} = iH_z, \qquad (5.5)$$

whence:

$$\psi' = \frac{\sqrt{2}}{2} \begin{pmatrix} E_x + iH_x \\ E_z + iH_z \\ E_z - iH_z \\ -E_x + iH_x \end{pmatrix}, \qquad (5.6)$$

Substituting these functions in the Dirac equation we will receive the correct Maxwell equations for the electromagnetic waves in double quantity. It is possible to assume, that the functions ψ' correspond to the electromagnetic wave, moving under the angle of 45 degrees to both coordinate axes.

Thus, from above it follows that *every choice of the Dirac matrices defines only the direction of the initial electromagnetic wave.* Obviously, this is a physical origin why “the physical consequences of Dirac’s equation do not depend on the special choice of Dirac’s matrices” (Fermi,1960).

6.0. The electromagnetic form of the EM electron theory Lagrangian

As a Lagrangian of the Dirac theory can take the expression (Schiff, 1955):

$$L_D = \psi^+\left(\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}} + \hat{\beta}\ mc^2\right)\psi, \qquad (6.1)$$

For the electromagnetic wave moving along the $-y$-axis the equation (6.1) can be written:

$$L_D = \frac{1}{c}\psi^+\frac{\partial\ \psi}{\partial\ t} - \psi^+\hat{\alpha}_y\frac{\partial\ \psi}{\partial\ y} - i\frac{mc}{\hbar}\psi^+\hat{\beta}\ \psi, \qquad (6.2)$$

Transferring each term of (6.2) in the electrodynamics form we obtain the electromagnetic form of the Dirac theory Lagrangian:

$$L_{DM} = \frac{\partial\ U}{\partial\ t} + div\ \vec{S} - i\frac{\omega}{4\pi}\left(\vec{E}^2 - \vec{H}^2\right), \qquad (6.3)$$

(Note that in the case of the variation procedure we must distinguish the complex conjugate field vectors $\vec{E}^*, \vec{H}^*$ and $\vec{E}, \vec{H}$). Using the complex electrical and "magnetic" currents (1.8') and (1.8'') we take:

$$L_{DM} = \frac{\partial\ U}{\partial\ t} + div\ \vec{S} - \left(\vec{j}^{\ e}\vec{E} - \vec{j}^{\ m}\vec{H}\right), \qquad (6.4)$$

It is interesting that since $L_s = 0$ thanks to (1.6), we can take the equation:

$$\frac{\partial\ U}{\partial\ t} + div\ \vec{S} - \left(\vec{j}^{\ e}\vec{E} - \vec{j}^{\ m}\vec{H}\right) = 0, \qquad (6.5)$$

which has the form of the energy-momentum conservation law for the Maxwell equation with current.

7.0. The Lorentz force expression in EM representation

According to our theory for the EM particles stability in the twirled waves (i.e. into the EM particles) the force must appear, which is perpendicular to the trajectory of motion of the EM fields. But in this case the tangential force (by our chose – along the y-axis), must absent, since it would provoke the tangential acceleration of the electron fields.

The expression of Lorentz's force by the energy-momentum tensor of electromagnetic field τ_μ^ν is well known (Tonnelat, 1959; Ivanenko and Sokolov, 1949). This tensor is symmetrical and has the following components:

$$f_\mu = -\frac{1}{4\pi}\frac{\partial\ \tau^\nu{}_\mu}{\partial\ x^\nu} \equiv -\frac{1}{4\pi}\partial_\nu\tau_\mu{}^\nu, \qquad (7.1)$$

Here, first three components describe the Lorentz force density vector, and fourth component corresponds to the energy conservation law.

Using (7.1) it can be written:

$$f_x = f_z = 0, \quad f_y \equiv -\left(\frac{\partial\ \vec{g}}{\partial\ t} + grad\ U\right), \tag{7.2}$$

$$f_4 = -\left(\frac{1}{c}\frac{\partial\ U}{\partial\ t} + c\ div\ \vec{g}\right), \tag{7.3}$$

As we see by using of the symmetrical energy-momentum tensor we don't obtain the needed components of the force since here $f(y) \neq 0$.

The right result can obtain using antisimmetrical spin tensor $\alpha_{\mu\nu}$ (3.2). Then we have:

$$f_\mu = -\frac{1}{4\pi}\frac{\partial\ \alpha_\mu^\nu}{\partial\ x^\nu} \equiv -\frac{1}{4\pi}\partial_\nu \alpha_\mu^\nu, \tag{7.4}$$

or:

$$\begin{cases} f_x = -\left(\dfrac{\partial\ \alpha_{12}}{\partial\ x_2} + \dfrac{\partial\ \alpha_{14}}{\partial\ x_4}\right) \\ f_y = 0 \\ f_z = -\left(\dfrac{\partial\ \alpha_{32}}{\partial\ x_2} + \dfrac{\partial\ \alpha_{34}}{\partial\ x_4}\right) \\ f_0 = 0 \end{cases}, \tag{7.5}$$

Using (1.6) and (3.2) we obtain of Lorenz's force components:

$$2\pi f_x = E_x\left(\frac{1}{c}\frac{\partial\ H_z}{\partial\ t} - \frac{\partial\ E_x}{\partial\ y}\right) + H_z\left(\frac{1}{c}\frac{\partial\ E_x}{\partial\ t} - \frac{\partial\ H_z}{\partial\ y}\right) +$$

$$+ H_x\left(\frac{\partial\ E_z}{\partial\ t} + \frac{\partial\ H_x}{\partial\ y}\right) + E_z\left(\frac{1}{c}\frac{\partial\ H_x}{\partial\ t} + \frac{\partial\ E_z}{\partial\ y}\right)$$

$$f_y = 0,$$

$$2\pi f_z = E_x\left(\frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y}\right) - H_z\left(\frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y}\right) +$$
$$+ H_x\left(\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y}\right) - E_z\left(\frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y}\right)$$
$$f_4 = 0, \tag{7.6}$$

For the linear photon all the brackets in (7.6) are equal to zero according to Maxwell's equation. It means that appear no forces in linear photon. When photon rolls up around any of the axis, which are perpendicular to the *Y*-axis, we will get the additional current terms.

If to take that the field vector of type $\vec{F} = \vec{F}(\vec{r})e^{-\omega t}$ describes geometrically the vectors rotation, we can for the twirled semi-photon write:

$$\frac{\partial E_x}{\partial t} = i\omega\ E_x + \frac{\partial E_x}{\partial t}, \tag{7.7'}$$

$$\frac{\partial E_z}{\partial t} = i\omega\ E_z + \frac{\partial E_z}{\partial t}, \tag{7.7''}$$

For spinning photon (E_x, H_z), the force components are (the upper left index shows the spinning axis *OZ* or *OX*).

$${}^z f_x = 2i\frac{1}{4\pi}\frac{\omega}{c}E_x(E_x + H_z) = 2\frac{1}{c}j_\tau \cdot (E_x + H_z), \tag{7.8}$$

for spinning photon (E_z, H_x):

$${}^x f_z = -2i\frac{1}{4\pi}\frac{\omega}{c}E_z(E_z - H_x) = -2\frac{1}{c}j_\tau \cdot (E_z - H_x), \tag{7.9}$$

$$f_y = 0, \tag{7.10}$$

$$f_4 = 0, \tag{7.11}$$

what corresponds to our representations about the dynamics of twirled semi-photon.

8.0. The equation of the ring EM wave field motion

We can suppose that 4-vector-potential of electromagnetic field, multiplied to the electron charge e, $\left\{e\varphi,\ \frac{e}{c}\vec{A}\right\}$ is the 4-vector of the energy-momentum of the curvilinear wave field $\{\varepsilon_p, \vec{p}_p\}$ (see chapter 2).

Therefore, the well-known analysis of Dirac's electron equation in the external field can be used for the analysis of the equations of the inner twirled photon field by the changes:

$$\frac{e}{c}\vec{A} \to \vec{p}_p, \; e\varphi \to \varepsilon_p, \; m \to 0, \tag{8.1}$$

As it is known (Akhiezer and Berestetskii, 1965; Schiff, 1955), the equation of the electron motion in the external field can be found from the next operator equation, having the Poisson brackets

$$\frac{d\hat{O}}{dt} = \frac{\partial \hat{O}}{\partial t} + \frac{1}{i\hbar}\left(\hat{O}\hat{\mathrm{H}} - \hat{\mathrm{H}}\hat{O}\right), \tag{8.2}$$

where $\hat{O}$ is the physical value operator, whose variation we want to find and $\hat{\mathrm{H}}$ is the Hamilton operator of Dirac’s equation.

The Hamilton's operator of the Dirac equation is equal (Schiff, 1955; Akhiezer and Berestetskii, 1965):

$$\hat{\mathrm{H}} = -c\hat{\vec{\alpha}}\,\hat{\vec{P}} - \hat{\beta}\, mc^2 + \varepsilon, \tag{8.3}$$

where $\hat{\vec{P}} = \hat{\vec{p}} - \vec{p}_p$ is full momentum of twirled photon.

For $\hat{O} = \hat{\vec{P}}$ from (8.3) we have:

$$\frac{d\hat{\vec{P}}}{dt} = \left[-grad\;(e\varphi) - \frac{e}{c}\frac{\partial \vec{A}}{\partial t}\right] + \frac{e}{c}\left[\vec{\upsilon} \times rot\;\vec{A}\right], \tag{8.4}$$

or, substitute $\vec{\upsilon} = c\hat{\vec{\alpha}}$, where $\vec{\upsilon}$ - velocity of the electron matter, we obtain:

$$\frac{d\vec{P}}{dt} = e\vec{E} + \frac{e}{c}\left[\vec{\upsilon} \times \vec{H}\right] = f_L, \tag{8.5}$$

Since for the motionless electron $\frac{d\hat{\vec{P}}_p}{dt} = 0$, the motion equation is:

$$\left(\frac{\partial \vec{p}_p}{\partial t} + grad\;\varepsilon_p\right) - \left[\vec{\upsilon} \times rot\;\vec{p}_p\right] = 0, \tag{8.6}$$

Passing to the energy and momentum densities

$$\vec{g}_p = \frac{1}{\Delta\tau}\vec{p}_p, \; U_p = \frac{1}{\Delta\tau}\varepsilon_p, \tag{8.7}$$

we obtain the equation of matter motion of twirled photon:

$$\left(\frac{\partial\ \vec{g}_p}{\partial\ t} + grad\ U_p\right) - \left[\vec{\upsilon} \times rot\ \vec{g}_p\right] = 0\,, \tag{8.8}$$

Let us analyse the physical meaning of (8.8), considering the motion equation of ideal liquid in form of Lamb's-Gromek's equation (Lamb, 1931). In this case, when the external forces are absent, this equation is:

$$\left(\frac{\partial\ \vec{g}_l}{\partial\ t} + grad\ U_l\right) - \left[\vec{\upsilon} \times rot\ \vec{g}_l\right] = 0\,, \tag{8.9}$$

where $U_l\,, \vec{g}_l$ -energy and momentum density of ideal liquid.

Comparing (8.8) and (8.9) is not difficult to see their mathematical identity. From this follows the interesting conclusion: the inner particles' equation may be interpreted as the motion equation of ideal liquid.

According to (8.5, 8.6) from (8.9) we have

$$\frac{\partial\ \vec{g}_p}{\partial\ t} + grad\ U_p = \vec{f}_L\,, \tag{8.10}$$

where f_L is the Lorenz force. As it is known the term $\left[\vec{\upsilon} \times rot\ \vec{g}_p\right]$ in (8.9) is responsible for centripetal acceleration. Probably, we have the same in (8.8). If the "photon' liquid" move along the ring of r_p radius, then the angular motion velocity ω is tied with $rot\ \vec{\upsilon}$ by expression:

$$rot\ \vec{\upsilon} = 2\vec{\omega}_p = 2\omega_p \vec{e}_z\,, \tag{8.11}$$

and centripetal acceleration is

$$\vec{a}_n = \frac{1}{2}\vec{\upsilon} \times rot\ \vec{\upsilon} = \frac{\upsilon^2}{r_p}\vec{e}_r = c\omega_p \vec{e}_r\,, \tag{8.12}$$

where $\vec{e}_r$ is unit radius-vector, $\vec{e}_z$ - is unit vector of *OZ*-axis. As a result the equation (5.25) has the form of Newton's law:

$$\rho\ \vec{a}_n = \vec{f}_L\,, \tag{8.13}$$

This result can be seen as the electromagnetic representation of the Erenfest theorem (Shiff,1955).

Conclusion

The above results proof that the non-linear EM representation of the Dirac theory give the classical explanations of all particularities of the Dirac electron theory, which nevertheless don’t contradict to the quantum interpretation.